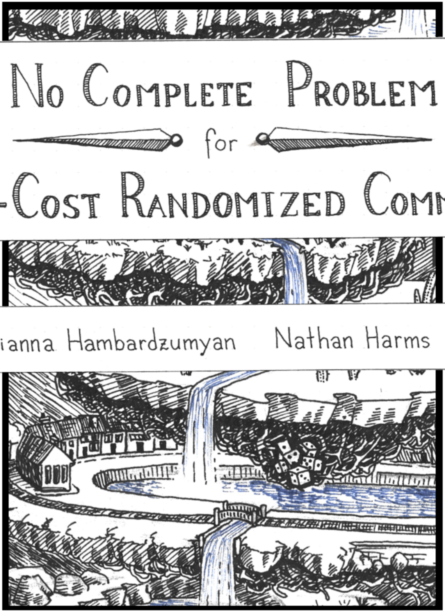
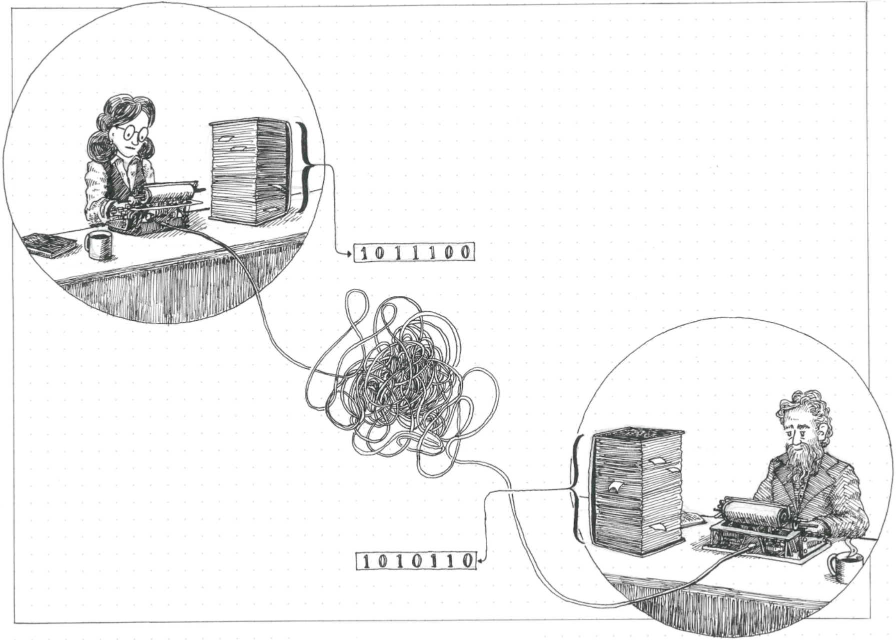
No Complete Problem for Constant-Cost Randomized Communication



Yuting Fang   Lianna Hambardzumyan   Nathan Harms   Pooya Hatami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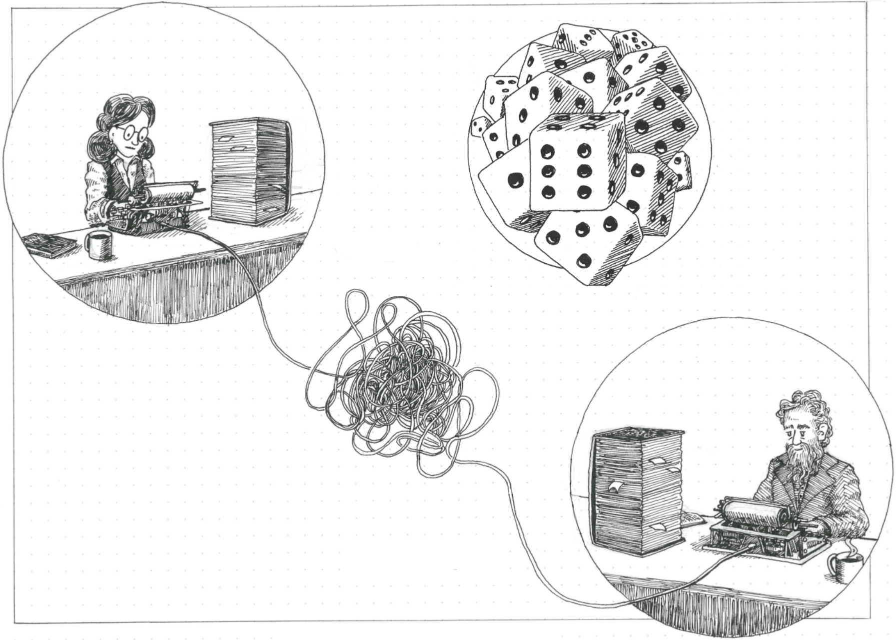
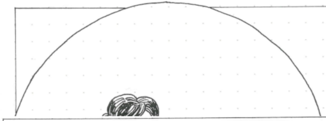
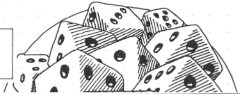




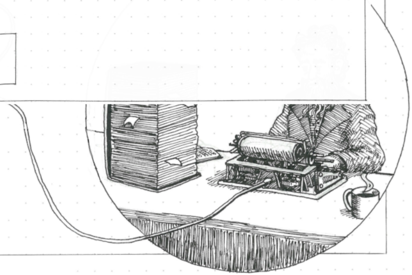


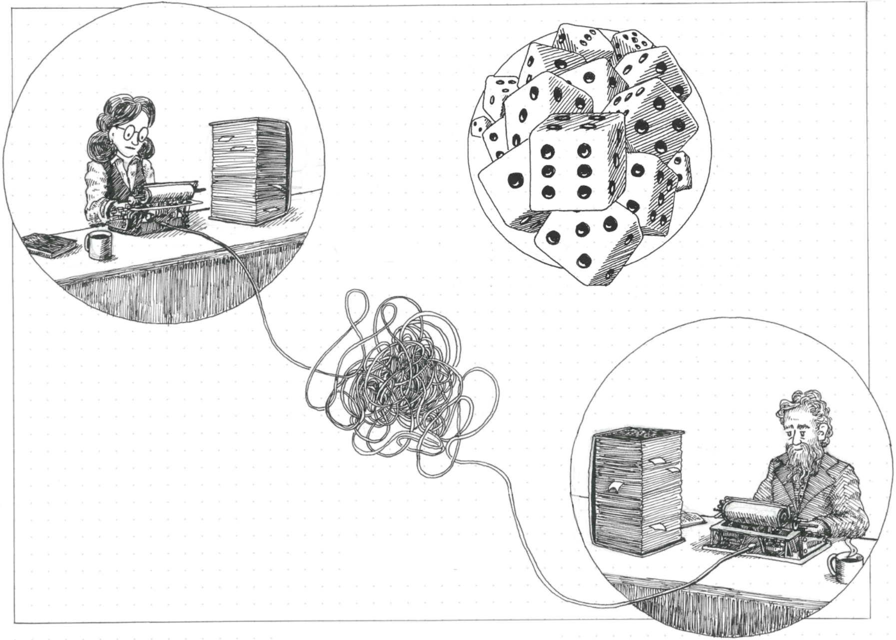
Why?

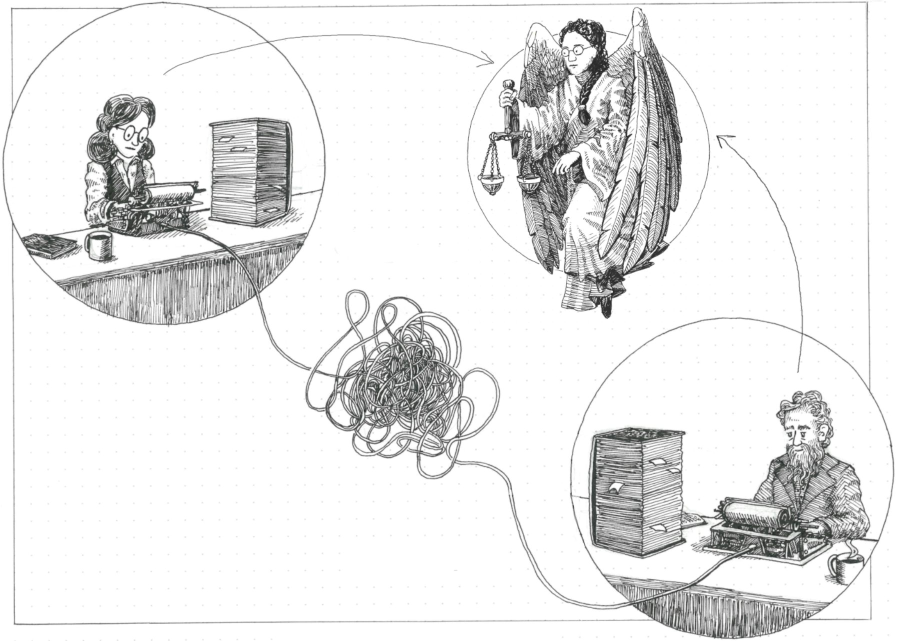
① Power of randomness
- Most extreme case
- Most basic lower bound
- "Fine-grained" understanding
- Standard techniques fail
- Complete characterization?
- Most evident structure

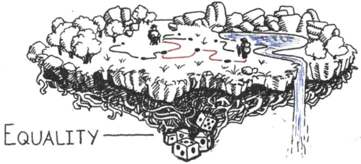
② Connections
- Matrix representations
- Structural graph theory
  - → New concepts
  - → New techniques
- Algebra
- Learning theory

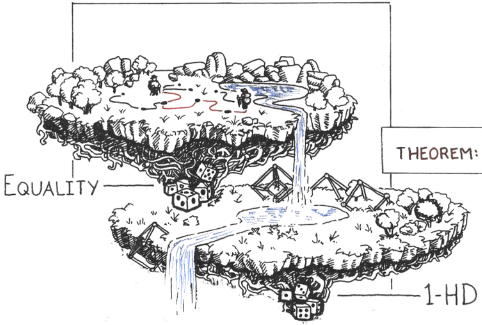
③ It is cool



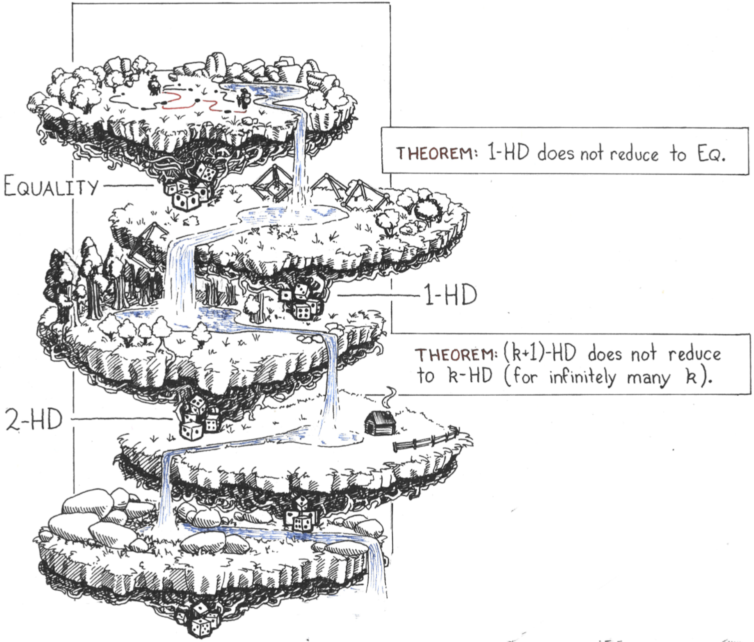





EQUALITY



THEOREM: 1-HD does not reduce to Eq.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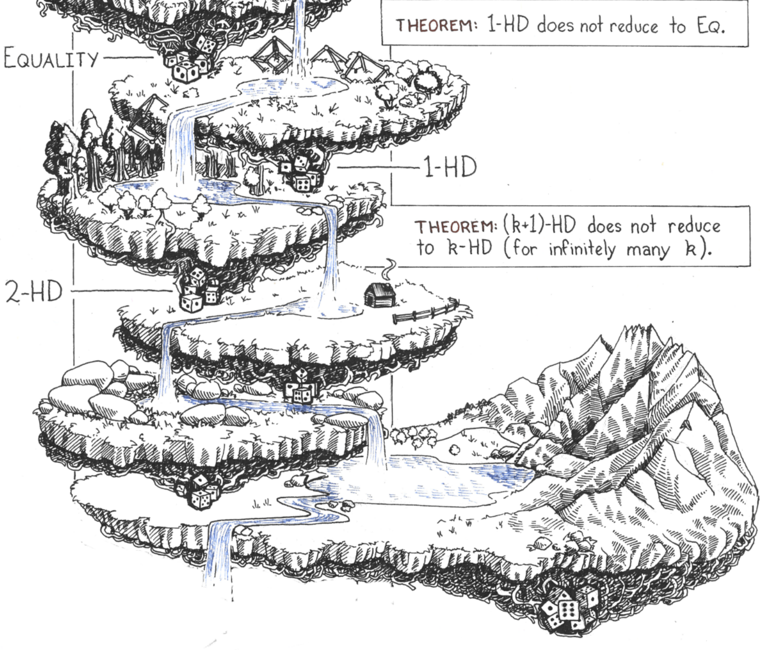
1-HD
THEOREM: (k+1)-HD does not reduce to k-HD (for infinitely many k).
2-HD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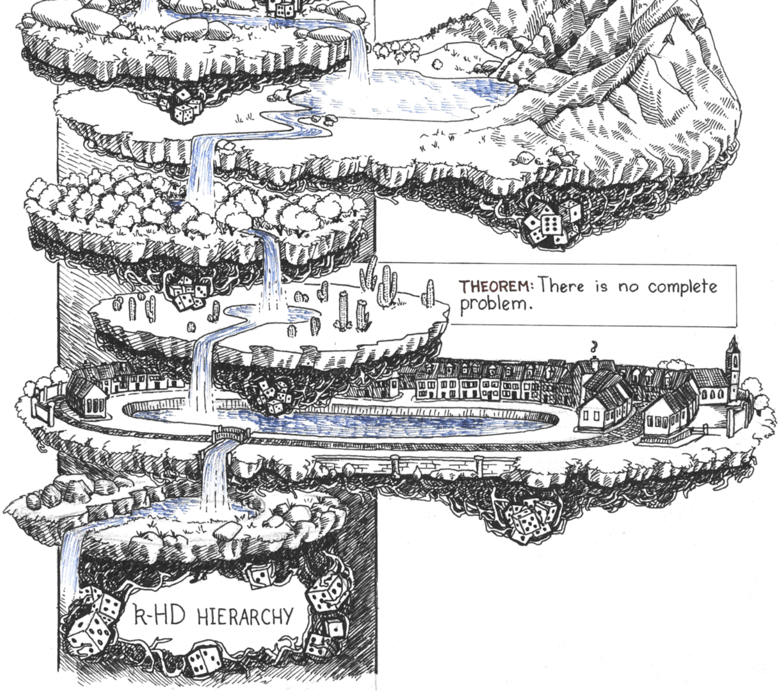
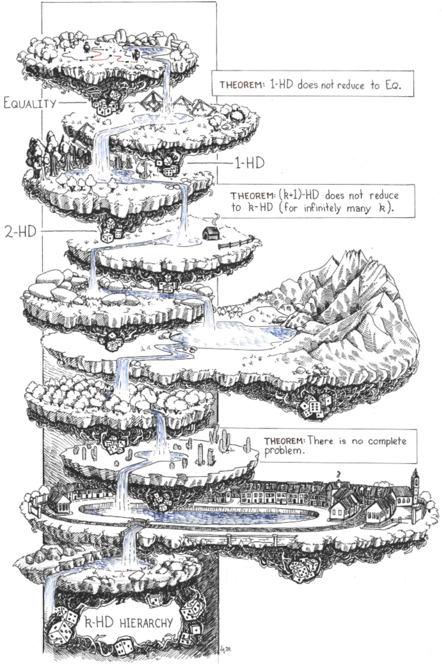

THEOREM: There is no complete problem.
$\mathfrak{k}$-HD HIERARCHY



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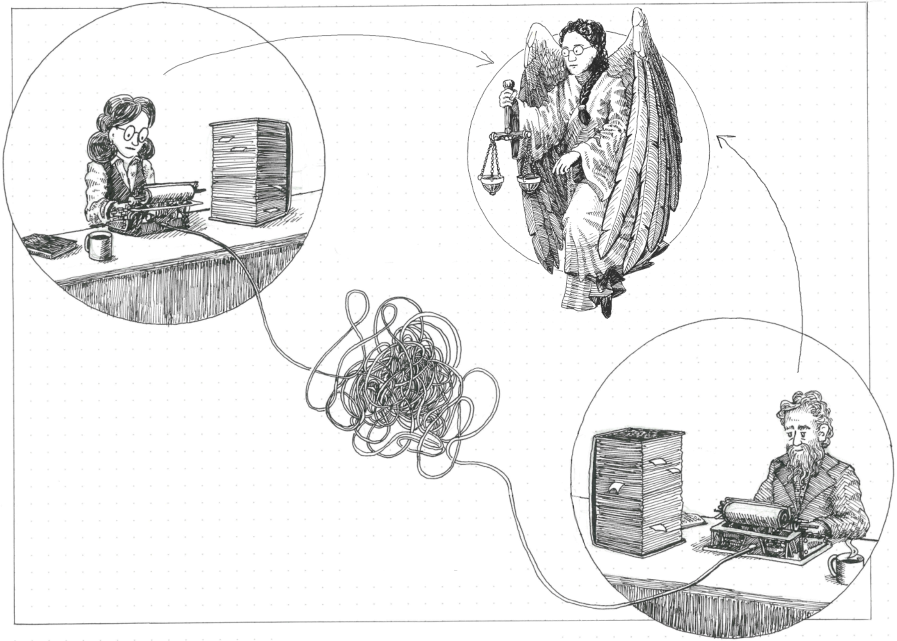

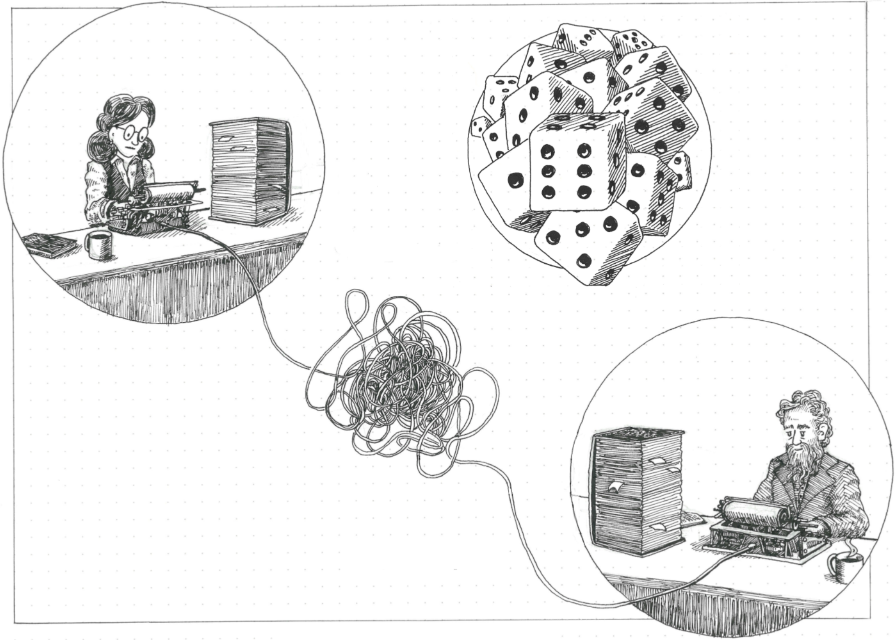

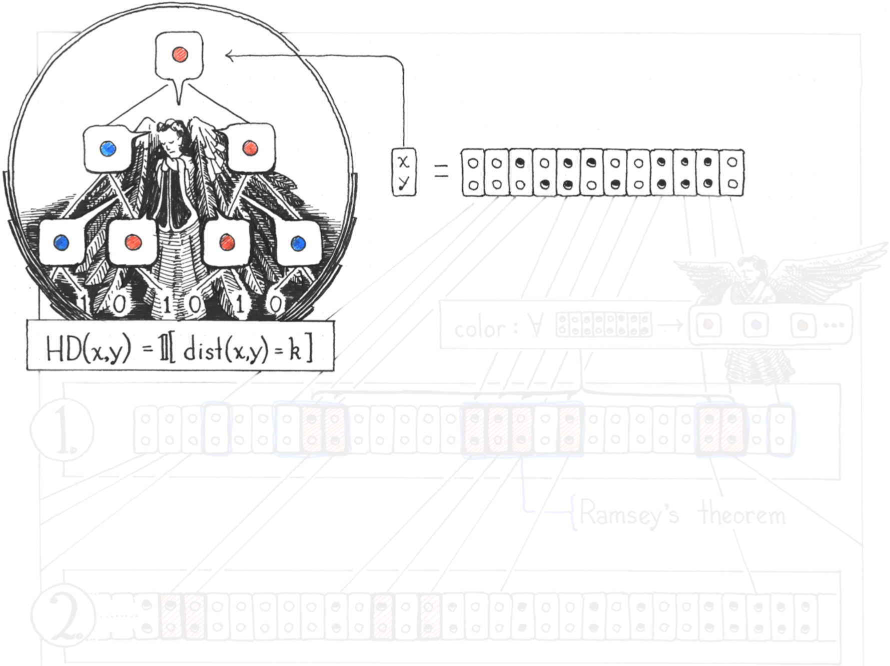
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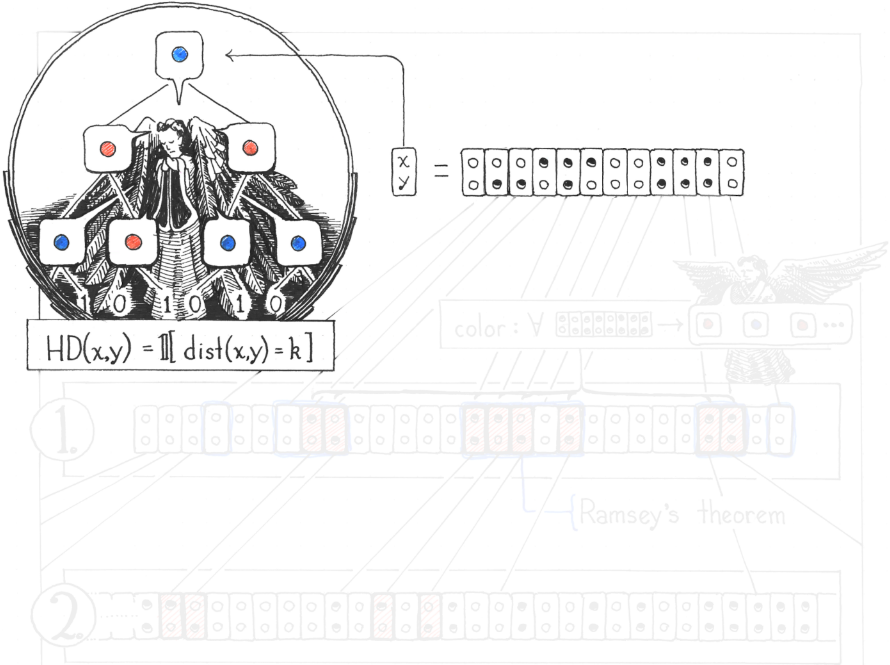
HD(x,y) = 𝟙[dist(x,y) = k]
x
y
=
color : ∀
1.
Ramsey's theorem
2.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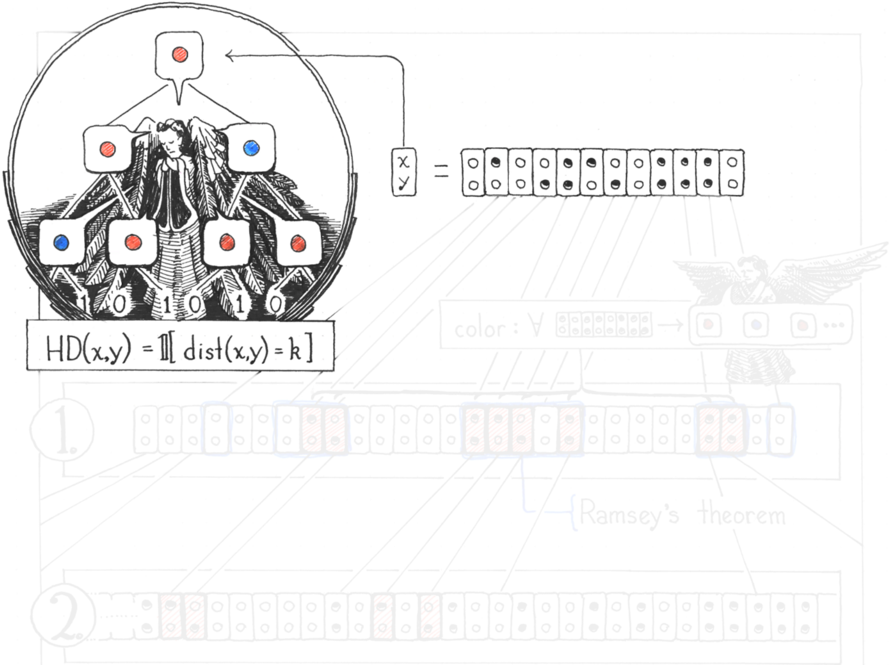

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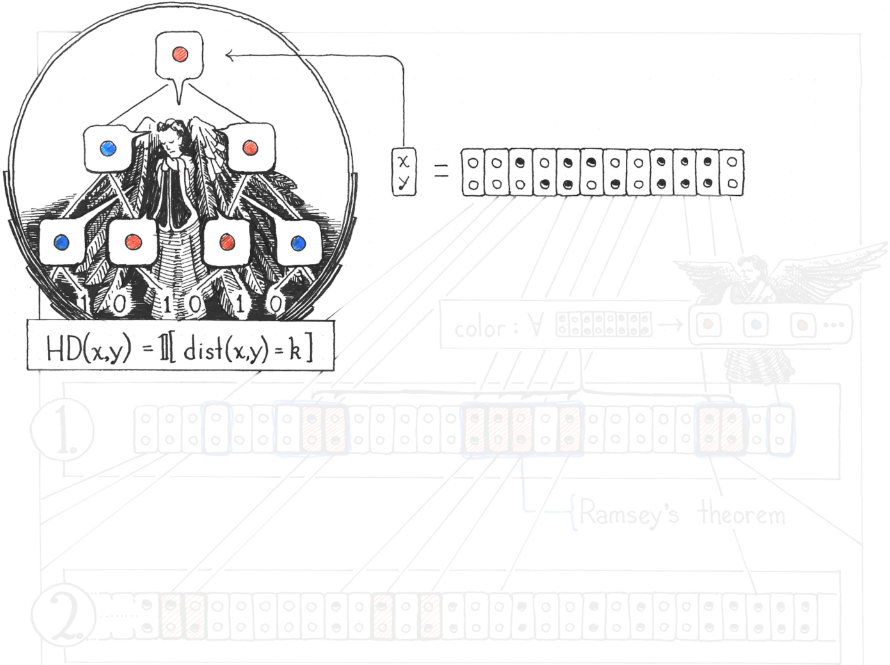

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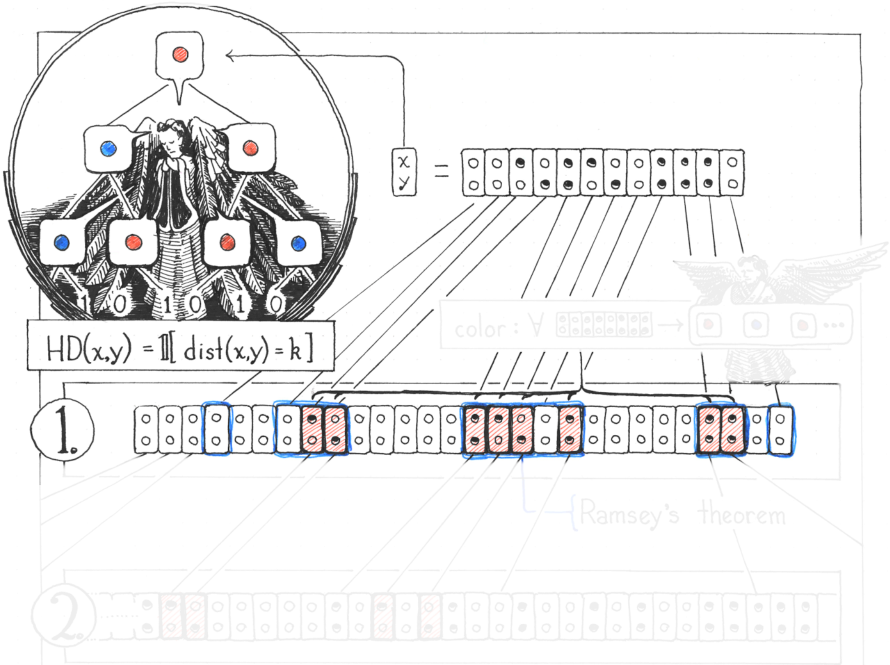

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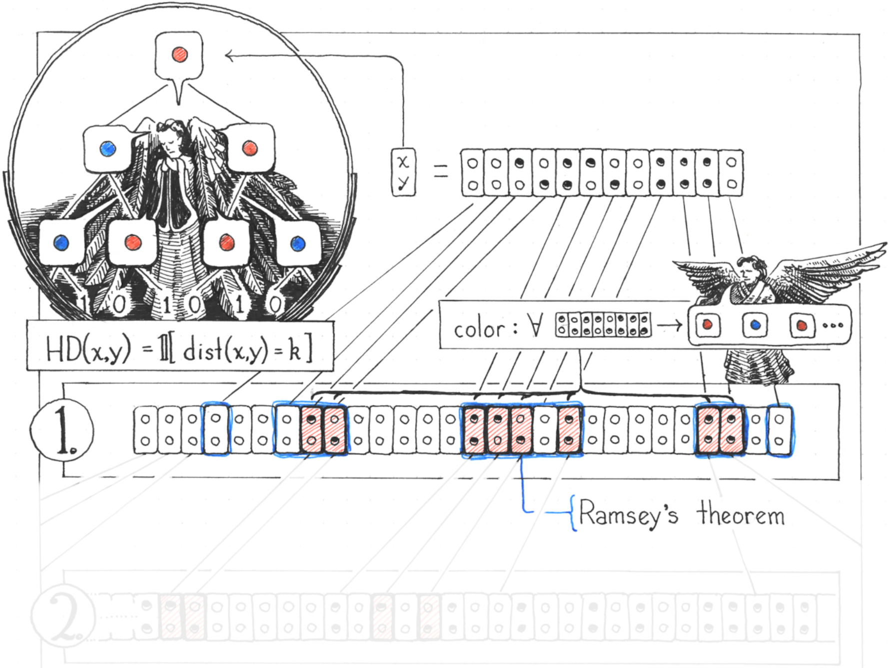

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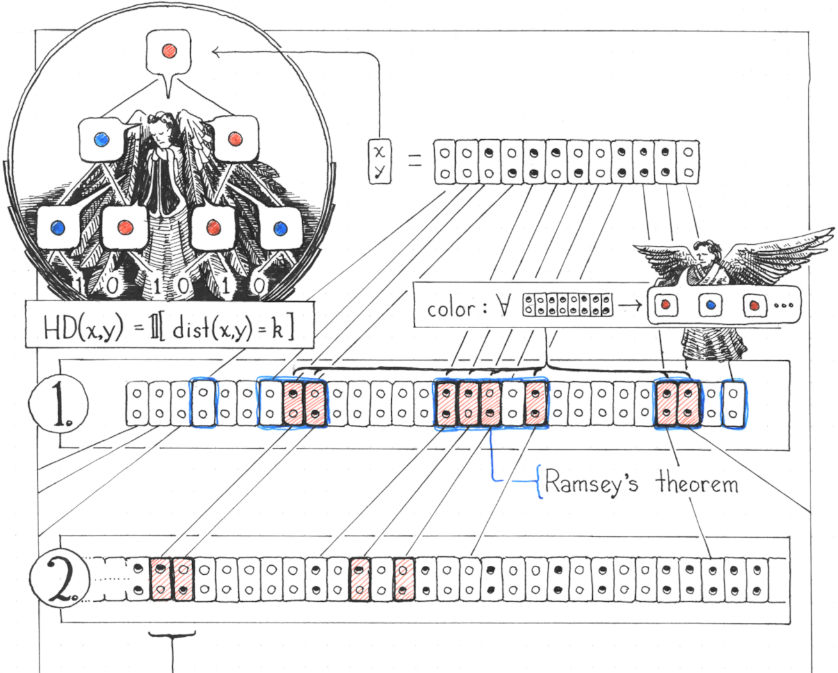

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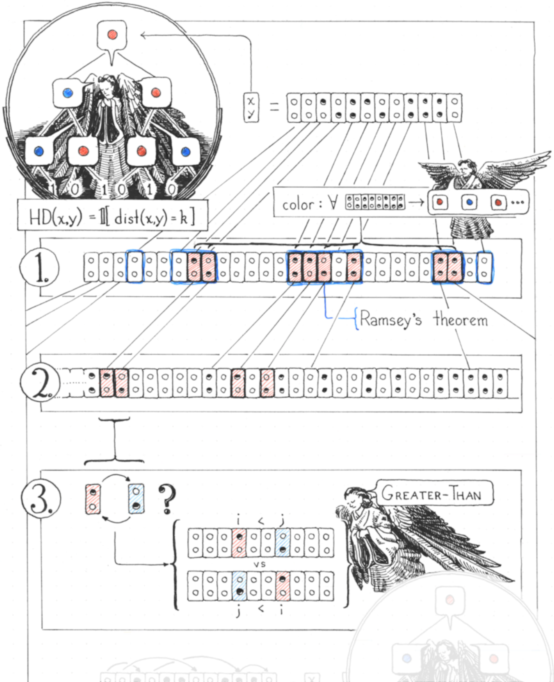
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color : ∀
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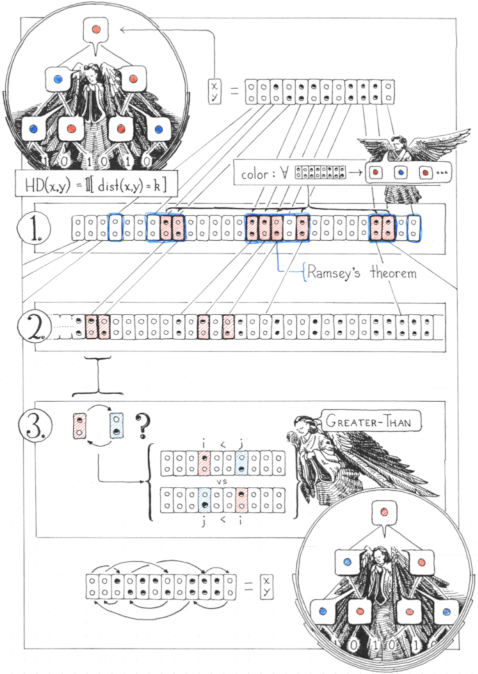
2.
3.
?
Greater-Than
i < j
vs
j < i



x
y
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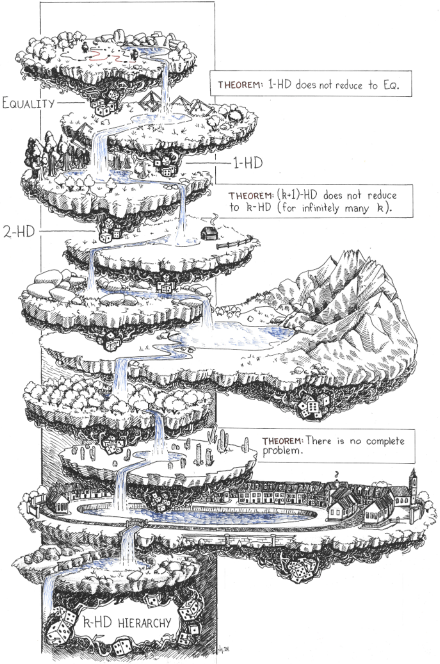
i < j
vs
j < i
GREATER-THAN
= x y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