Optimal adjacency labels for subgraphs of Cartesian products

Louis Esperet Nathan Harms Viktor Zamaraev

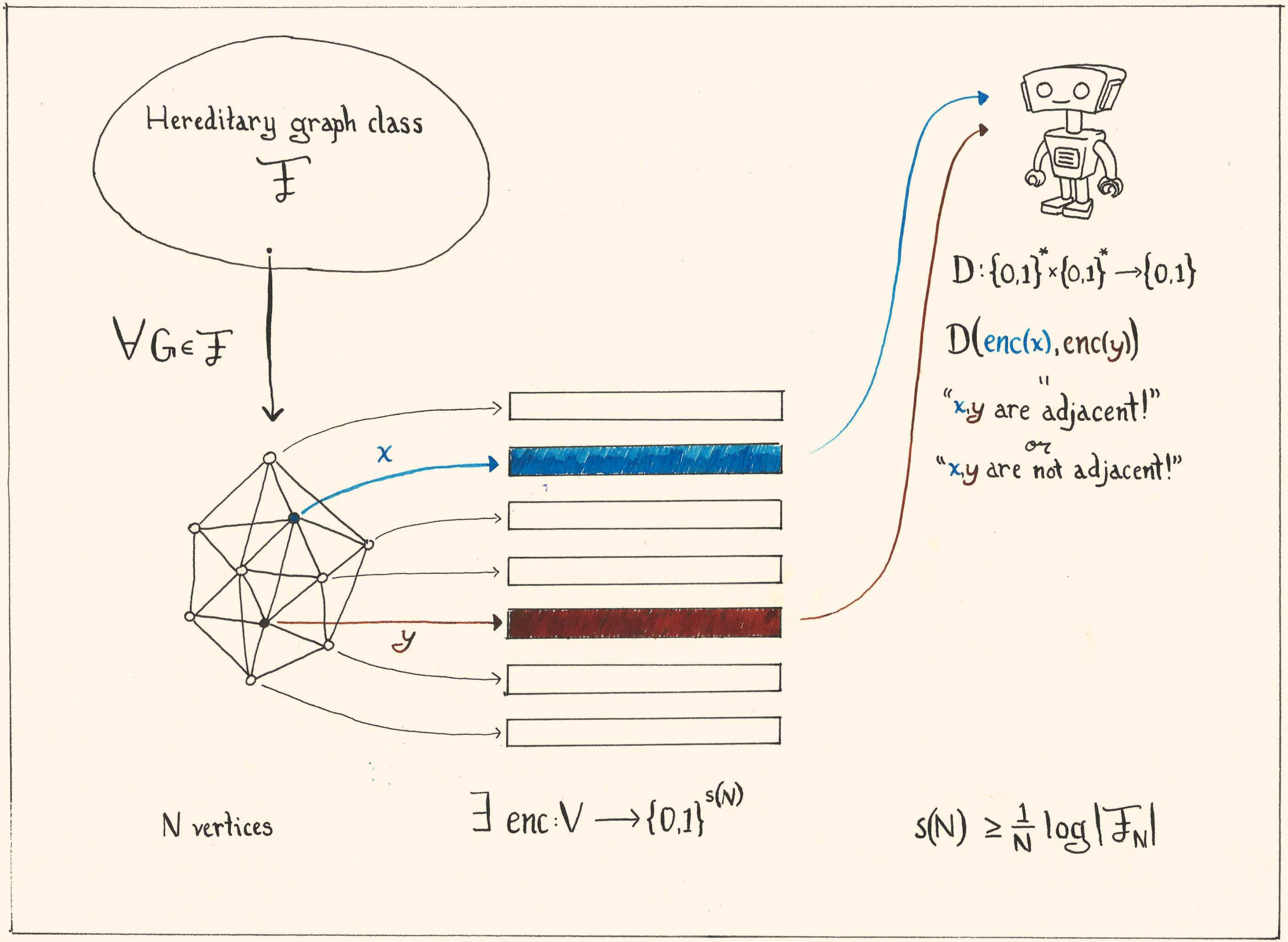
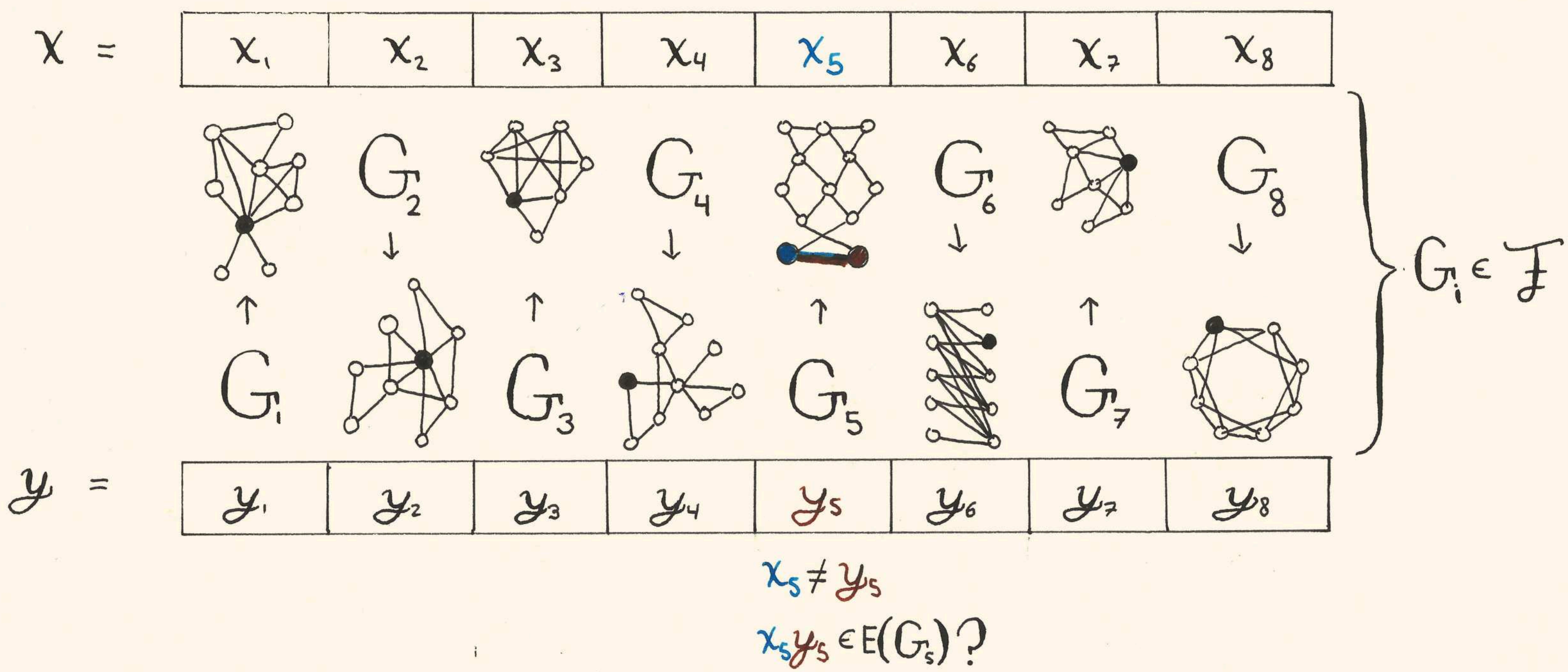


fig 1. Adjacency Labelling

$$\mathcal{F}^{\square} : \{ G = \underbrace{G_1 \square G_2 \square \cdots \square G_d}_{\mathcal{F}} \, , \; d \in \mathbb{N} \}$$



$her(\mathcal{F}^{\square})$	Delete vertices	$mon(\mathcal{F}^{\square})$	Delete vertices & edges

fig 2. Cartesian Products

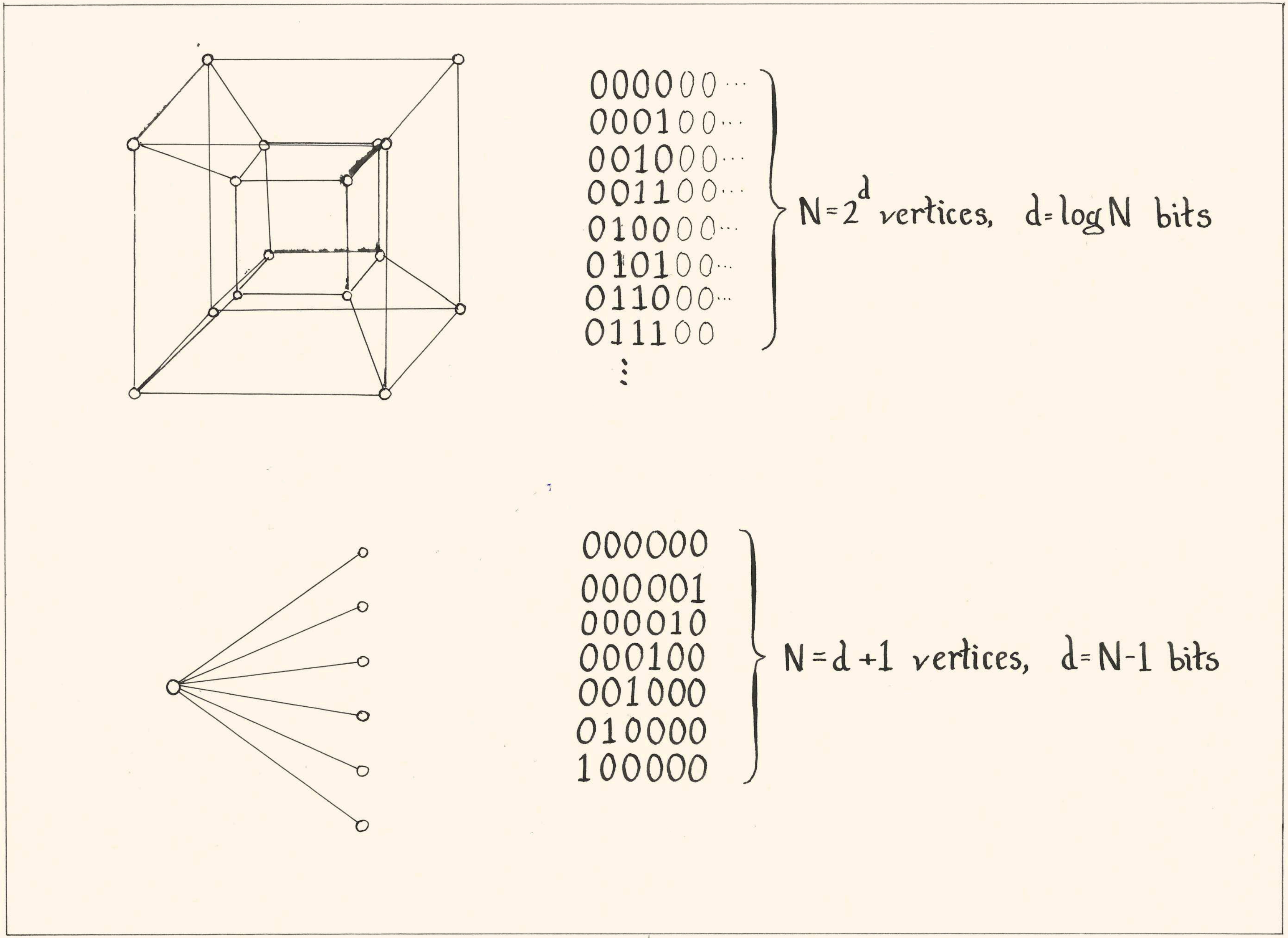


fig 3. Hypercubes

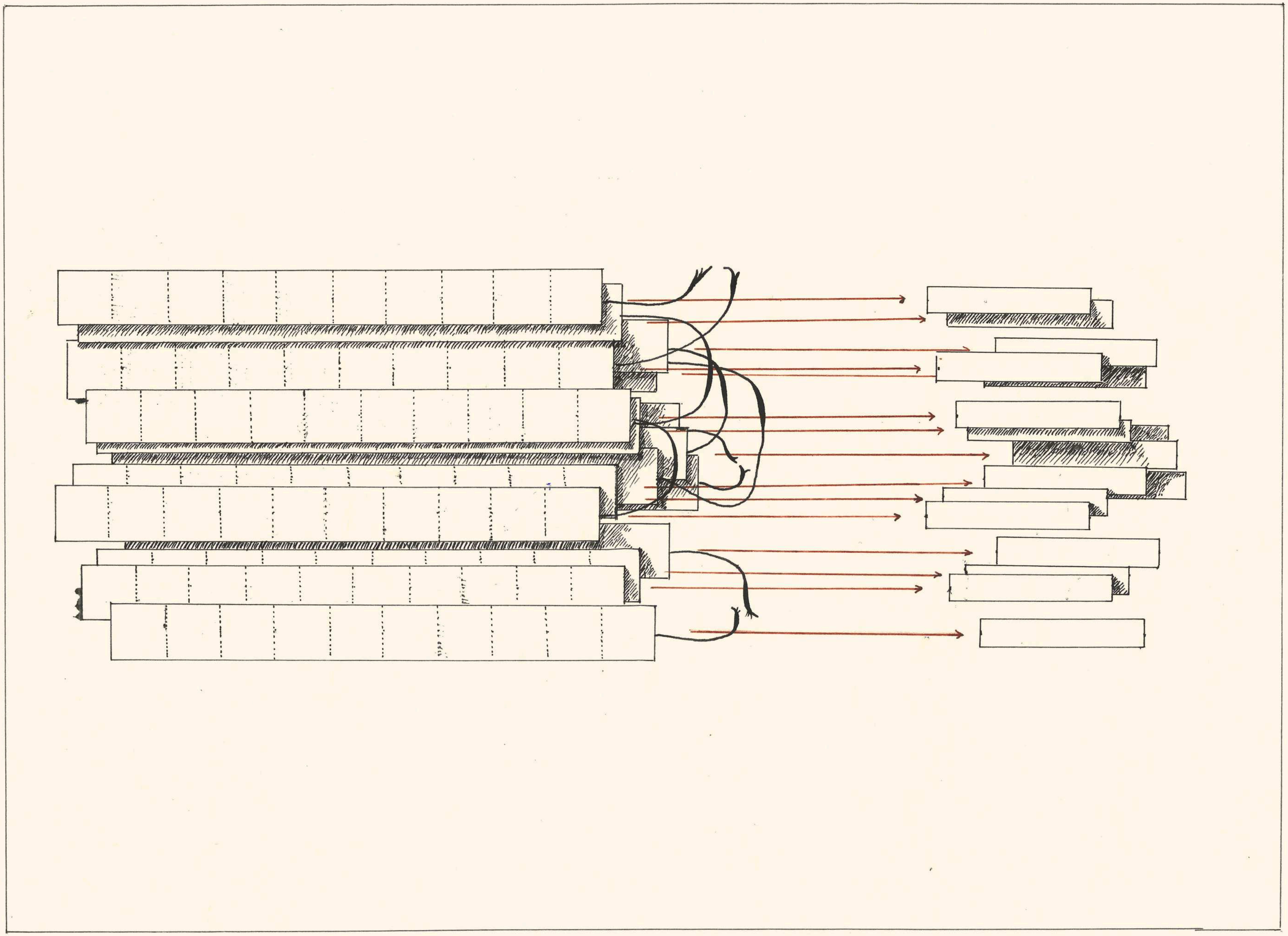


fig. 4. Adjacency Labels for Cartesian Products

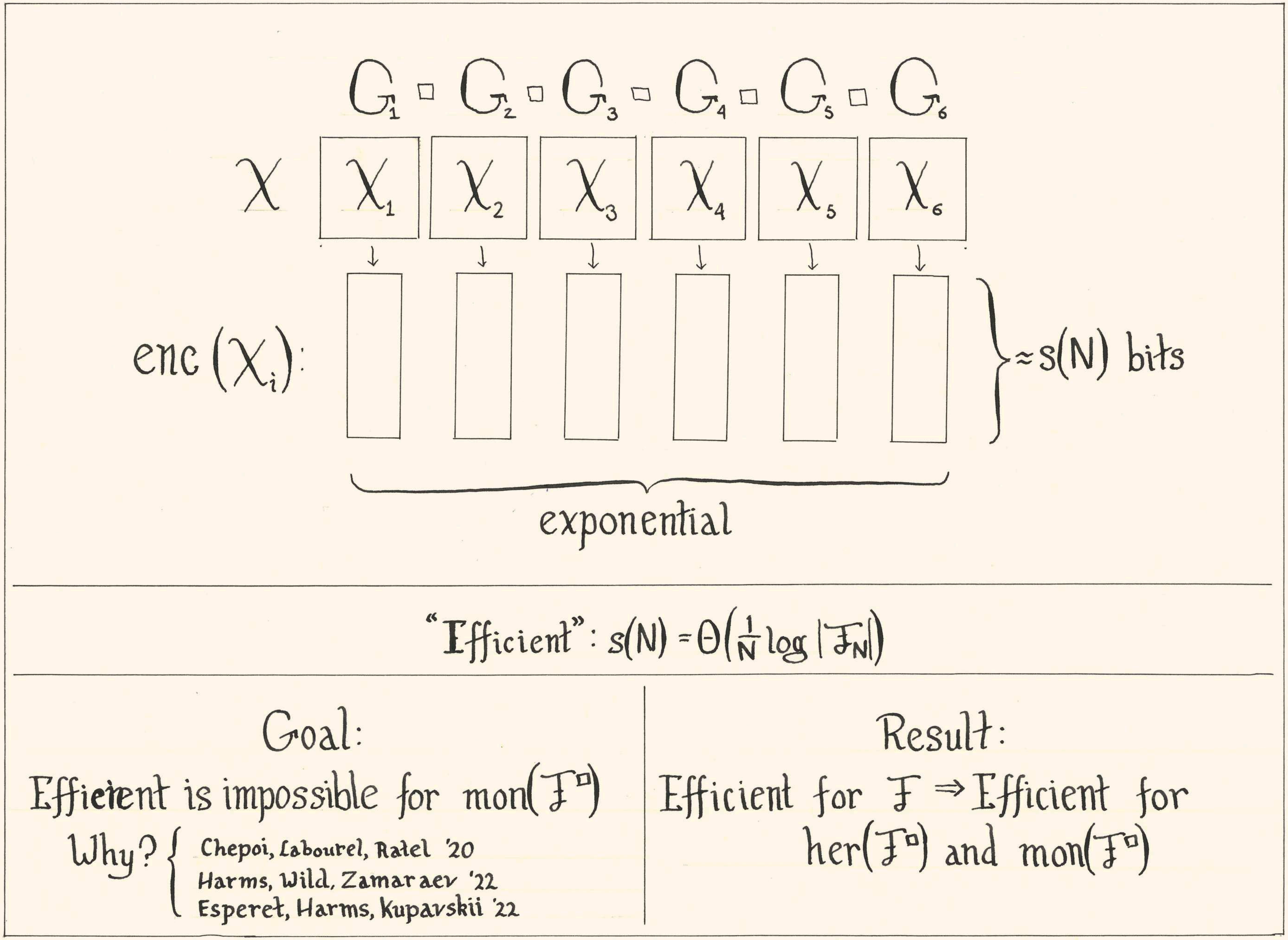
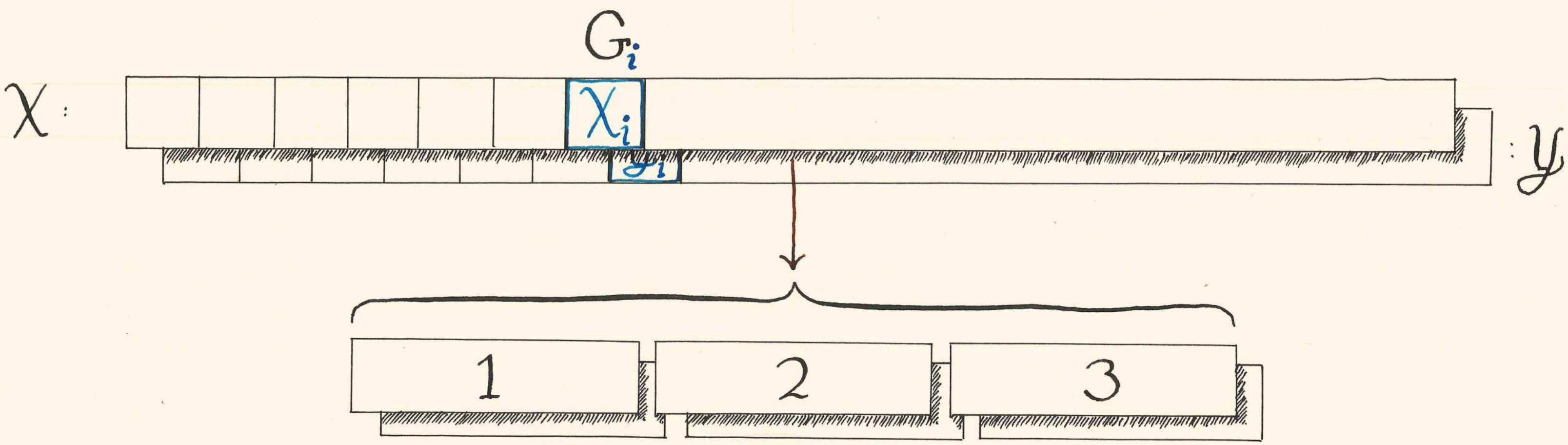
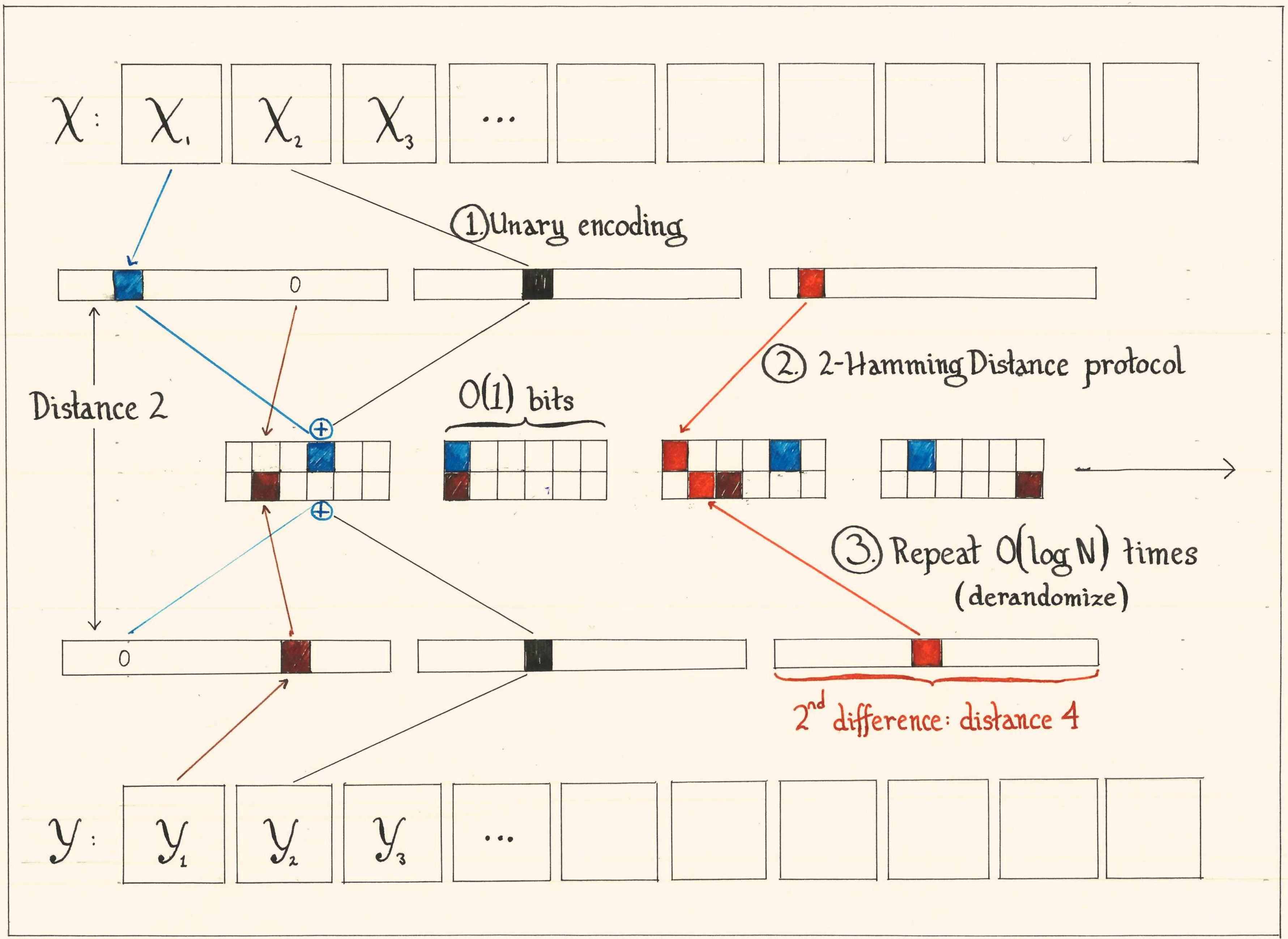


fig 5. Results



Phase 1	Phase 2	Phase 3
x, y differ in only one coordinate, i	$x_i y_i \in E(G_i)$	$xy \in E(G)$ deleted?
Randomized Communication	Additive Combinatorics	Perfect Hashing

fig 6. THE PLAN



X :
X_1
X_2
X_3
...
① Unary encoding
0
Distance 2
② 2-Hamming Distance protocol
O(1) bits
③ Repeat O(log N) times
(derandomize)
0
2nd difference: distance 4
Y :
Y_1
Y_2
Y_3
...

fig 7. PHASE I.

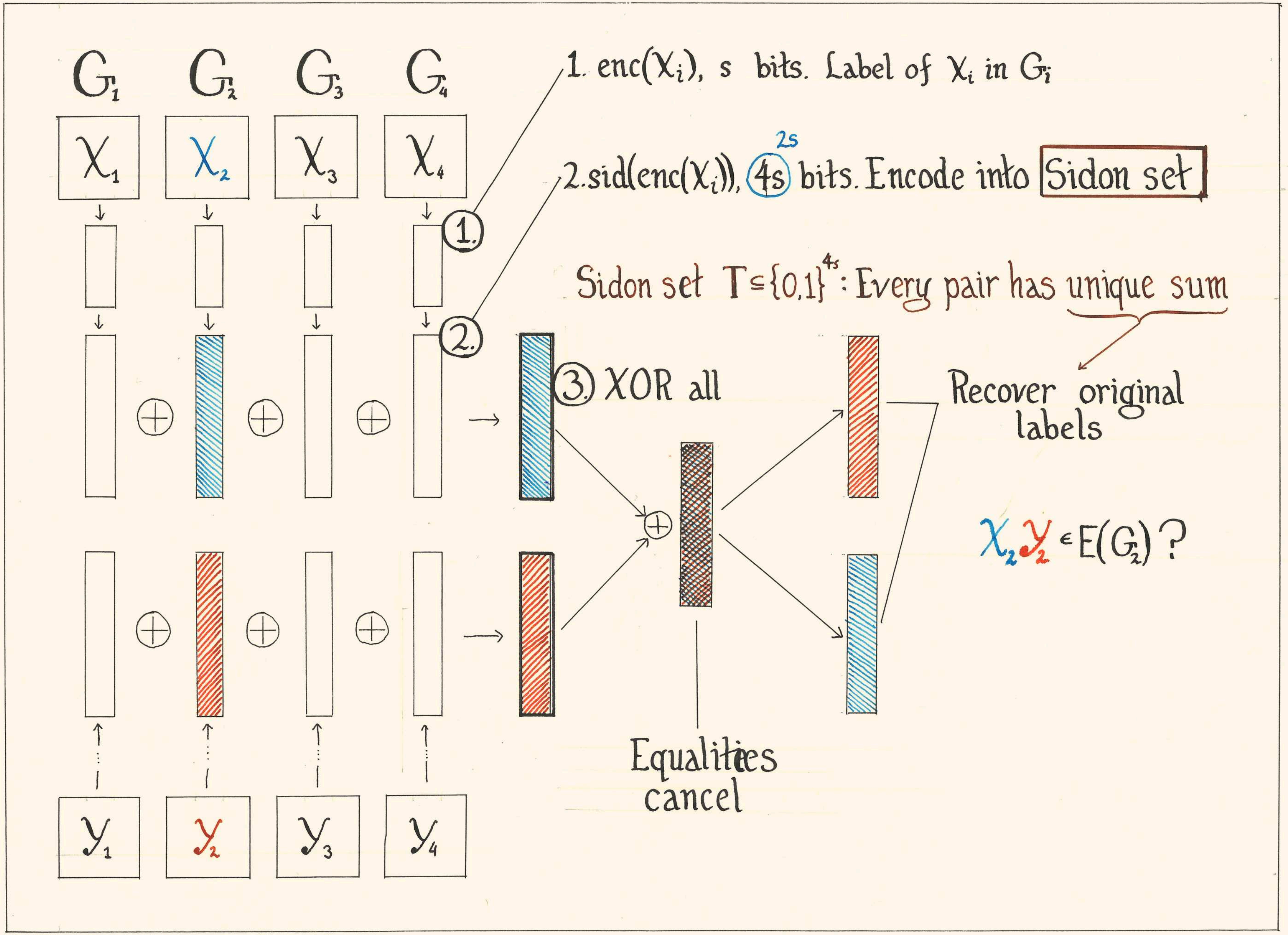
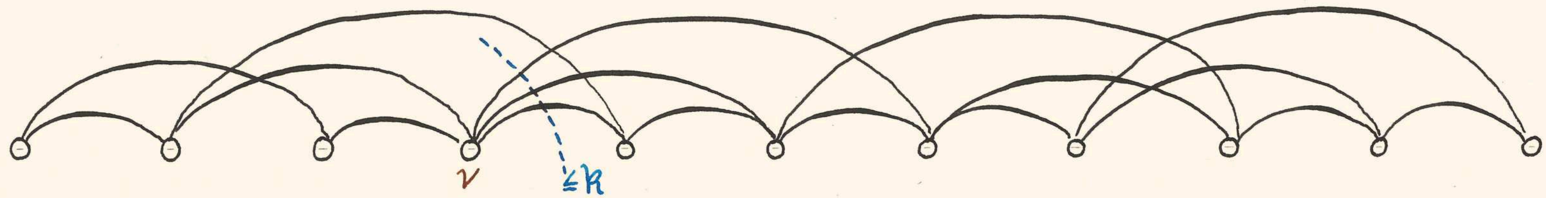


fig 8. PHASE II



Degeneracy k: at most k forward neighbors F_v (Hypercube: $k = \log N$)

Minimal perfect hash: $h_v : F_v \to [k]$, stored in $O(k + \log\log N)$ bits

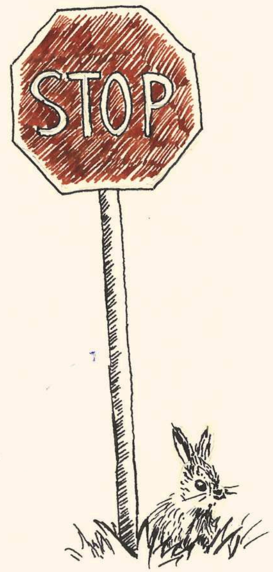
$\log N$	$k + \log\log N$	k
Order	Hash	Deletions in F_v

Total size:

$$\underbrace{\log N}_{\text{I.}} + \underbrace{S(N)}_{\text{II.}} + \underbrace{k + \log N}_{\text{III.}}$$

(Label size for $\mathcal{F}$)

fig 9. PHASE III.


STOP